Quantum Computing Meets Quantum Chemistry:

A Potential New Era of Simulation and Study

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Wallenberg Centre for Quantum Technology

Take-home messages – Big picture

• What is quantum chemistry? Why is it worthwhile?



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• What is quantum computing? What are the potential advantages?





Take-home messages – Big picture

• What is quantum chemistry? Why is it worthwhile?

• What is quantum computing? What are the potential advantages?

- How can it help quantum chemistry?
 - What are state-of-the-art approaches?





Werner Dobrautz



The Case for Quantum Computing

Quantum Computing for Quantum Chemistry

- Conclusion and Outlook

Surprisingly small systems at the center of fascinating physical and chemical effects

Motivation: Haber-Bosch process and biological nitrogen fixation

Haber-Bosch Process



- Crucial for fertilizer production
- 2% of world's energy consumption
- 3% of global carbon emissions
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Haber-Bosch Process 180 bar

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- Ambient pressure and temperatures
- Process not yet understood → Bio-catalysts for more efficient and greener ammonia production

Problem: Strongly correlated quantum systems

- Small molecular systems act as catalysts: Iron-Molybdenum cofactor (FeMoCo)
- Experimental study very difficult!



Problem: Strongly correlated quantum systems

- Small molecular systems act as catalysts: Iron-Molybdenum cofactor (FeMoCo)
- Experimental study very difficult!
- → Numerical studies of relevant **electronic** quantum phenomena necessary!





Quantum Chemistry – Applications



- Drug discovery
- Materials design
- Battery development
- ...

Nitrogen fixation

Artificial photosynthesis

High-T_c superconductivity

Quantum Chemistry – Applications



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Nitrogen fixation Artificial photosynthesis

High-T_c superconductivity

Accurate theoretical understanding at quantum-scale for bottom-up materials design!

 $\approx 30\%$ of high-performance computing resources for chemistry-related problems

Insight on **physical** and **chemical properties** (ground- and excited state energies, chemical reactions, ...) of quantum systems by **solving the Schrödinger equation:**

 $\hat{H} \left| \Psi \right\rangle = E \left| \Psi \right\rangle$

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Current state of all electrons described by the wavefunction: $\Psi(\mathbf{r}_1, \mathbf{r}_2, \dots \mathbf{r}_n)$

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All information of a quantum system contained in electronic Hamiltonian:

$$\hat{H} = \hat{T}_{\mathsf{Kin.}}(\mathbf{r}) + \hat{V}_{\mathsf{Attr.}}(\mathbf{r}, \mathbf{R}) + \hat{V}_{\mathsf{Rep.}}(\mathbf{r}, \mathbf{r}')$$



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 $\label{eq:coulomb} \begin{array}{l} \mbox{Coulomb repulsion correlates all electrons of a system} \to \mbox{analytic solution too} \\ \mbox{complex} \to \mbox{approximations and computational approaches} \end{array}$

Quantum Chemistry – Accuracy and cost

Depending how accurately we treat correlation: various methods and **levels of theory** to solve the Schrödinger equation



Cytochrome c: enzyme that eliminates toxic radicals produced by cells. From Santagati et al., arXiv:2301.04114

Quantum Chemistry – Accuracy and cost

Depending how accurately we treat correlation: various methods and **levels of theory** to solve the Schrödinger equation



Need highly-accurate methods to describe strongly correlated problems



















Number of possible states for given number of electrons and orbitals

Mol.	#orbitals	#electrons	#states
H_2	2	2	4
LiH	4	4	36
Be_2	8	8	4900
H_2O	12	12	$\sim 8\cdot 10^5$
C_2H_4	16	16	$\sim 16\cdot 10^6$
F_2	18	18	$\sim 2 \cdot 10^9$

All possible excitations from HF state

 \approx 256 GB to store wavefunction

Quantum Chemistry meets Quantum Computing

We have the equations at hand, but exponentially costly on classical computers!





Quantum Chemistry meets Quantum Computing

Quantum computers could provide a potential **speedup!**





The Case for Quantum Computing

Classical bit

Quantum bit = qubit

$a \left| 0 \right\rangle + b \left| 1 \right\rangle$

Quantum bit = qubit

 $\begin{aligned} a & |0\rangle + b & |1\rangle \\ & |a|^2 + |b|^2 = 1 \end{aligned}$

Qubits – **Bloch Sphere**



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Qubits – Bloch Sphere



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Bringing **two** qubits together:

$$|\Psi\rangle = \overbrace{(|0\rangle + |1\rangle)}^{\text{qubit 1}} \otimes \overbrace{(|0\rangle + |1\rangle)}^{\text{qubit 2}} = |00\rangle + |01\rangle + |10\rangle + |11\rangle \qquad \text{4 states}$$

Multiple Qubits – Entanglement

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Three qubits:

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 8 states

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N qubits can encode exponentially many (2^N) states. 40 qubits enough to encode the $\sim 2 \cdot 10^9$ states of F₂! \rightarrow Need new **quantum algorithms** to use this potential advantage!



Quantum Computing for Quantum Chemistry

How can Quantum Computing help Quantum Chemistry?



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- Map our problem (Hamiltonian/basis functions) onto quantum hardware/qubits
 - Qubits encode occupation of spin-orbitals $\in [0, 1]$

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- Map our problem (Hamiltonian/basis functions) onto quantum hardware/qubits
 - Qubits encode occupation of spin-orbitals $\in [0,1]$
- $\rightarrow\,$ Use quantum algorithms for ground-, excited states, dynamics, \ldots

1) Prepare an initial state $|\Phi_0\rangle$:

 $|\Phi_0\rangle = \begin{pmatrix} 1\\1\\0\\0 \end{pmatrix}$



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2) Perform unitary operations of chosen quantum algorithm:

$$|\Phi\rangle = \hat{U} |\Phi_0\rangle = a_1 \begin{pmatrix} 0\\0\\0\\0 \end{pmatrix} + a_2 \begin{pmatrix} 1\\0\\0\\0 \end{pmatrix} + \dots + a_{2^N} \begin{pmatrix} 1\\1\\1\\1 \end{pmatrix}$$

1) Prepare an initial state $|\Phi_0\rangle$:

3) Measure observable $\langle \hat{O} \rangle$



2) Perform unitary operations of chosen quantum algorithm:

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$$|\Phi_0\rangle = \begin{pmatrix} 1\\1\\0\\0 \end{pmatrix}$$

 $\sim 15 {\rm mK}$ Magnetic shielding









. . .



Effect of noise:

 $\sim 15 \mathrm{mK}$

- Bit flip: $|0\rangle \leftrightarrow |1\rangle$
- Phase flip: $|0\rangle \leftrightarrow |0\rangle$
- Decoherence: $|0\rangle + |1\rangle \rightarrow |0\rangle + e^{i?} |1\rangle$







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• . . .





Use many physical qubits to encode a logical qubit:

11111	$\rightarrow 1$	$11011 \rightarrow 1$	

 $00000 \to 0 \qquad \qquad 01000 \to 0$



Unitary op. Phase $\hat{U}|\Psi
angle={
m e}^{i heta}|\Psi
angle$ Eigenstate

Unitary op. Phase
$$\hat{U}|\Psi
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$$\hat{H} |\Psi\rangle = E |\Psi\rangle$$

$$\downarrow$$

$$\mathbf{e}^{-i\hat{H}t} |\Psi\rangle = \mathbf{e}^{-iEt} |\Psi\rangle$$



No matrix diagonalization! Subroutine of Shor's algorithm

$$\hat{H} |\Psi\rangle = E |\Psi\rangle$$

$$\downarrow$$

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Probabilities



No matrix diagonalization! Subroutine of Shor's algorithm

Schrödinger eq. $\hat{H} |\Psi\rangle = E |\Psi\rangle$ $e^{-iHt} |\Psi\rangle = e^{-iEt} |\Psi\rangle$ 0.8 $\sum_{i} c_i$ 0.6 0.4 0.2 046 0.0



No matrix diagonalization! Subroutine of Shor's algorithm

$$\hat{H} |\Psi\rangle = E |\Psi\rangle$$

$$\downarrow$$

$$e^{-i\hat{H}t} |\Psi\rangle = e^{-iEt} |\Psi\rangle$$

- Many qubits, deep circuits \rightarrow requires error corrected quantum devices
- State preparation: how to get good approximations of $|\Psi\rangle$?

Transition toward fault-tolerance

NISQ:

- Noisy and small quantum devices
- Limited utility
- Hybrid approaches

Fault-tolerant QC: Quantum advantage

- Shor's algorithm
- Quantum phase estimation

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Continuous transition to fault-tolerant QC:

- Develop intuition on quantum algorithm development
- Transferability of developed algorithms to FT regime
- Feedback for experimentalists to improve devices
- Near-term utility and relevant applications
- No need for 'quantum for everything'

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Fault-tolerant QC: Quantum advantage

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Near-term approaches and our work

NISQ Era – Hybrid Quantum-Classical Approach



- Use short-depth quantum circuits that fit current hardware
- Improve on classical estimates by non-classical states
- Store quantum state with exponentially fewer resources

NISQ Era – Hybrid Quantum-Classical Approach



NISQ Era – Hybrid Quantum-Classical Approach



State-of-the-art

State-of-the-art – Quantum Computing enhanced Quantum Monte Carlo

Quantum-enhanced QMC methods:

- Use the QPU to alleviate computational bottlenecks of conventional QMC methods



State-of-the-art – Quantum Computing Quantum Monte Carlo





Classical and quantum trial wave functions in auxiliary-field quantum Monte Carlo applied to oxygen allotropes and a CuBr, model system \oslash

Maximilian Amsler ©; Peter DegImann ©; Matthias Degroote ©; Michael P. Kaicher ©; Matthew Kiser ©; Michael Kühn ©; Chandan Kumar ©; Andreas Maler ©; Ceorgy Samsonidze ©; Anna Schroeder ©; Michael Streif ⊠; Davide Vodola ©; Christopher Wever ©; CUTAC Material Science Working Group

State-of-the-art – Subspace Expansion Methods

$$\hat{H} \left| \Psi \right\rangle = E \left| \Psi \right\rangle \qquad \longrightarrow \qquad \mathsf{span} \{ \left| \psi \right\rangle, \hat{H} \left| \psi \right\rangle, \hat{H}^2 \left| \psi \right\rangle, \dots \}$$

State-of-the-art – Subspace Expansion Methods



2.0

AH/E2,10"

AH/E2×11

AH/E2×18

25

Our work

Use benefits of both quantum and classical resources



Algorithms:

Quantum imaginary time evolution (QITE)

- Classical optimization
- Resource reduction: Qubits and circuit depth
- Error mitigation



- Algorithms:
 - Quantum imaginary time evolution (QITE)
- Classical optimization
- Resource reduction: Qubits and circuit depth
- Error mitigation










WD, Luo, Alavi, PRB 99 (7), 075119 (2019); Cohen, Luo, Guther, WD, Tew, Alavi, JCP 151 (6), 061101 (2019).



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$$\hat{H} |\Psi\rangle = E |\Psi\rangle \quad \rightarrow \quad |\Psi\rangle = f |\Phi\rangle \quad \rightarrow \quad \overbrace{f^{-1}\hat{H}f}^{\hat{H}_{\mathrm{TC}}} |\Phi\rangle = E |\Phi\rangle$$

WD, Luo, Alavi, PRB 99 (7), 075119 (2019); Cohen, Luo, Guther, WD, Tew, Alavi, JCP 151 (6), 061101 (2019)



$$\hat{H} |\Psi\rangle = E |\Psi\rangle \quad \rightarrow \quad |\Psi\rangle = f |\Phi\rangle \quad \rightarrow \quad \overbrace{f^{-1}\hat{H}f}^{\dot{H}_{\rm TC}} |\Phi\rangle = E |\Phi\rangle$$

 $|\Phi
angle$ easier to represent with less basis functions/qubits ightarrow immense resource reduction

WD, Luo, Alavi, PRB 99 (7), 075119 (2019); Cohen, Luo, Guther, WD, Tew, Alavi, JCP 151 (6), 061101 (2019)

Quantum Computing – Resource Reduction – Transcorrelation



Smaller basis \rightarrow fewer qubits



Quantum Computing – Resource Reduction – Transcorrelation



Quantum Computing – Resource Reduction – Transcorrelation



Towards real chemical accuracy on current quantum hardware through the transcorrelated method, J. Chem. Theory Comput. 20, 10, 4146 (2024) W. Dobrautz, I. O. Sokolov, K. Liao, P. Lopez Rios, M. Rahm, A. Alavi, I. Tavernelli



Reducing quantum circuit depth for noise-resilient quantum chemistry, E. Magnusson, A. Fitzpatrick, S. Knecht, M. Rahm, W. Dobrautz, Faraday Discussions on *Correlated Electronic Structure* (2024)

Reference-State Error Mitigation: A Strategy for High Accuracy Quantum Computation of Chemistry, *J. Chem. Theory Comput.*, **19**, 3, 783 (2023) P. Lolur, M. Skogh, **W. Dobrautz**, C. Warren, J. Biznárová, A. Osman, G. Wendin, J. Bylander, M. Rahm



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Quantum Computing – Algorithms and Classical Optimization

Orders of magnitude increased accuracy for quantum many-body problems on quantum computers via an exact transcorrelated method, *Phys. Rev. Research* **5**, 023174 (2023), I. O. Sokolov^{*}, **W. Dobrautz^{*}**, H. Luo, A. Alavi, I. Tavernelli



Quantum Computing – Algorithms and Classical Optimization

Energy $|\mathbf{r}_i - \mathbf{r}_j|$ Qubits $\hat{U}(\boldsymbol{\theta}_{0})$ Full WF $|\Psi\rangle = e^{\hat{J}} |\Phi\rangle$ Transcorr, WF $10^{(}$ error [t] 10^{-1} Absolute energy 10^{-3} 10^{-3} Momentum space - QITE/SV TC momentum space - QITE/SV momentum space - VOE/SV 10^{-2} 10^{-5} Number of layers Number of lavers

no-TC

31

- TC

 $|\Phi({\mathbf{r}})\rangle$

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Cost Plateau Local Minima Classical Parameter value computer: --Ground state -ONG (block-diag) - aBang 1. Prep. system 1. Metric: - Adam - aBroyden - gBang (block-diag) 2. Solve for: $\mathbf{A}_{\mathrm{ij}} = \langle oldsymbol{\partial}_{ heta_{\mathrm{i}}} \mathbf{\Phi} | oldsymbol{\partial}_{ heta_{\mathrm{i}}} \mathbf{\Phi}
angle$ lavers: 4 $\dot{\theta} = -\mathbf{A}^{-1}\mathbf{C}$ (b) $|\psi(\theta)|H|\psi(\theta)\rangle$ L(0, 0,) $L(\theta_1, \Theta_2)$ \Rightarrow 0.10^{2} 10 Number of circuit evaluations

Optimizing Variational Quantum Algorithms with qBang: Efficiently Interweaving Metric and Momentum to Tackle Flat Energy Landscapes, D. Fitzek, R. S. Jonsson, W. Dobrautz, C Schäfer, Quantum 8, 1313 (2024)

Conclusion and Outlook

Conclusion



Conclusion



Artificial photosynthesis

High-T_c superconductivity

Quantum Hardware Roadmaps











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Quantum Hardware Roadmaps



Quantum Error Correction





Quantum Error Correction of Qudits Beyond Break-even



Acknowledgments



Thank you for your attention!