# Chemistry Meets Quantum Computing: A New Era of Simulation and Study

### SmallTalks [about Nanoscience]

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### Outline

- **Part I:** Motivation take-home messages
  - Computational chemistry
  - The case for quantum computing
- **Part II:** Motivation: Quantum chemistry
  - Reducing hardware requirements with the **transcorrelated** method
  - Applications: Reducing the number of qubits for chemistry
  - Conclusions and outlook

# Learning goals:

- What is **computational chemistry**?
- Why is it worthwhile?
- Why do we need computers for chemistry?
- Why and how could **quantum computing** help?

Why is it worthwhile? Surprisingly small molecules responsible for fascinating physical and chemical effects

### Nitrogen fixation

Conversion of molecular nitrogen,  $N_2$ , to ammonia,  $NH_3$ ,  $\rightarrow$  important for fertilizers Haber-Bosch process: **1-2% of global energy consumption**, huge CO<sub>2</sub> emission





Iron-sulfur clusters

Artificial nitrogen fixation for cheaper and cleaner ammonia production for fertilizers

Conversion of solar energy into chemical energy





Manganese-Calcium-Oxygen Clusters

Artificial photosynthesis for carbon capture and hydrogen and oxygen for fuel cells

### Scientific method

- We want a **theoretical understanding** of these phenomena and explain the physical and chemical behavior of these systems
- We want to derive a mathematical description/model
- We want to test the validity of these models and compare with experiments
- We want to predict properties of new materials based on our models

 $\checkmark$  Mathematical description possible! Schrödinger equation

2 Too complex and impossible to solve exactly/analytically! Need computers to **approximately** solve the equations!  $\rightarrow$  computational chemistry.

 $\sim 30\%$  of high-performance computing used for chemistry-related problems

Computational chemistry

## How do we describe these phenomena? Atoms – the building blocks of nature



- If we knew the **position** and **motion** of all electrons and nuclei **at all times** we could describe the systems behavior and properties.
- Due to the size of the particles → quantum effects. We can not know the position and motion/momentum exactly at the same time. Heisenberg uncertainty relation/particle-wave duality.
- We can use **quantum mechanics** and write down an equation to describe the behavior of the system!

### Ingredients – what do we need?

We want an equation to describe the behavior of our system. How it evolves with time.

- 1. Nuclei **much heavier** than electrons (1000 fold)  $\rightarrow$  assume nuclei fixed!
- 2. Motion of electrons  $\rightarrow$  kinetic energy
- 3. Negative charged electrons attracted to positive charged nuclei
- 4. Negative charged electrons repel each other! 

  ✓ movement and position of all electrons depend on each other! It is a correlated problem!







### Schrödinger equation

Ingredients to describe the behavior of the electrons of our system

$$\hat{H} = T_{\text{Kinetic}}(\mathbf{r}) + V_{\text{Attraction}}(\mathbf{r}, \mathbf{R}) + V_{\text{Repulsion}}(\mathbf{r}, \mathbf{r}')$$



Current state of all the electrons at time t described by the so-called wavefunction:

$$\Psi(\mathbf{r}_1,\mathbf{r}_2,\ldots\mathbf{r}_n;t)$$

Schrödinger equation describes the change of the system with time t:

$$i \frac{\Psi(\mathbf{r},t)}{\partial t} = \hat{H} \Psi(\mathbf{r},t)$$

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## The case for quantum computing

### Problem: Nature is very complex

Cytochrome c: enzyme that eliminates toxic radicals  $(O_2^-, H_2O_2)$  produced by cells



From: arXiv:2301.04114

### How is information stored and handled on a computer?





- Modern electronics are based on transistors, which can be seen as electronic switches that are either "off" or "on"; 0 or 1.
- $\rightarrow$  A logical state with two possible values: a bit.
- These **bits** of information can either store data (RAM/harddrive) or control operations (CPU)



CPU

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### Quantum Bits – Qubits







3 qubit device @Chalmers

@ Chalmers

Qubits can be both  $|0\rangle$  and  $|1\rangle$ .

Qubits rely on quantum effects  $\rightarrow$  very fragile, easily influenced by environmental effects/noise. Need to isolate and cool them close to absolute zero! Only few of them...

Kantz et al.. Applied Physics Reviews 6, 021318



Quantum bits (qubits) can be in a superposition of both  $|0\rangle$  and  $|1\rangle$ 

Bringing together  $\mathbf{two}$  qubits:

$$|\Psi\rangle = \overbrace{(|0\rangle + |1\rangle)}^{\text{qubit 1}} \otimes \overbrace{(|0\rangle + |1\rangle)}^{\text{qubit 2}} = |00\rangle + |01\rangle + |10\rangle + |11\rangle \qquad 4 \text{ states}$$

Three qubits:

$$\begin{split} |\Psi\rangle &= \overbrace{(|0\rangle + |1\rangle)}^{\text{qubit 1}} \otimes \overbrace{(|0\rangle + |1\rangle)}^{\text{qubit 2}} \otimes \overbrace{(|0\rangle + |1\rangle)}^{\text{qubit 3}} \\ &= |000\rangle + |001\rangle + |010\rangle + |100\rangle + |011\rangle + |101\rangle + |110\rangle + |111\rangle \quad 8 \text{ states} \end{split}$$

n qubits can encode exponentially many  $(2^n)$  states.

 $\rightarrow$  Need new **quantum algorithms** to use this potential advantage!

## Part II

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### Ab Initio Quantum Chemistry – Electronic Structure Theory

All necessary information of a quantum system contained in electronic molecular Hamiltonian (Born-Oppenheimer approx. and atomic units)



**Electronic properties:** Ground- and low-lying excited state properties, energy differences, polarization, response functions, ...

<u>**Target:**</u> High / "chemical accuracy" to ensure predictability, interpretability and comparison with experimental results

Task: Solve the Schrödinger equation derived from first principles:

$$\hat{H} |\Psi(\mathbf{r}_1, \dots, \mathbf{r}_n)\rangle = E_0 |\Psi(\mathbf{r}_1, \dots, \mathbf{r}_n)\rangle$$

### Ab Initio Quantum Chemistry – Electronic Structure Theory

We have to choose a numerical **basis/orbitals** to perform our calculations in! In comp. chemistry: orbitals are smooth, "atomic-like" orbitals (for each atom):



For accurate results we need **dozens to hundreds** of orbitals per atom!

#### Problems for accurate description: Cusp condition

**Cusp condition:** Singularity of Coulomb potential,  $\frac{1}{r_{ij}}$ , for  $r_{ij} = 0$  $\rightarrow$  sharp cusp of exact wavefunction  $\Psi(\{\mathbf{r}\})$  at electron coalescence  $(r_{ij} = 0)$ 

$$\hat{H} = -\sum_{I,j} \frac{Z_I}{|\mathbf{R}_I - \mathbf{r}_j|} - \frac{1}{2} \sum_i \nabla_{\mathbf{r}_i}^2 + \frac{1}{2} \sum_{i \neq j} \frac{1}{|\mathbf{r}_i - \mathbf{r}_j|}, \quad \hat{H} |\Psi(\{\mathbf{r}\})\rangle = E_0 |\Psi(\{\mathbf{r}\})\rangle$$



Kato, Communications on Pure and Applied Mathematics 10 (2), 151 (1957)

### Digital Quantum Simulation/Computation

- 1. Map fermionic Hamiltonian onto quantum hardware/qubits
- 2. Use quantum algorithms to solve the problem at hand

**Circuit model:** Lines/wires represent qubits encoding the **occupation** (0-1) of the associated spin-orbital. Similar to classical circuits (AND, OR, ...) we can act with **operations/gates** on one or more qubits. For quantum computers have to be **reversible/unitary operations**  $\hat{U}(\theta) \rightarrow$  prepare **entangled** target state  $|\Psi(\theta)\rangle$ .



State preparation  $|\Psi(\boldsymbol{\theta})\rangle$ 

## Development Roadmap

#### IBM Quantum



Chalmers next-gen chip: 25 qubits, compared to **billions** of transistors in current CPUs! Only small problems currently tractable on quantum devices

### **Overview:** Quantum Chemistry on Quantum Computers

Current quantum hardware has many problems still: noise, decoherence and limited **number of qubits** – noisy intermediate-scale quantum (NISQ) era



#### Hybrid quantum-classical approach:

- Use pros of both classical and quantum hardware
- Use short-depth quantum **circuits** that fit current hardware
- Can improve on classical estimates by non-classical states
- Store quantum state with exponentially fewer resources

Need two qubits per orbital  $\rightarrow$  only very small, inaccurate chemistry calculations possible!

Reducing hardware requirements with the transcorrelated method

#### Cusp Condition – Explicitly Correlated Ansatz



Describe the cusp exactly and capture part of correlation with a **correlated (Jastrow)** wavefunction Ansatz

$$|\Psi({\mathbf{r}})\rangle = e^{\hat{J}} |\Phi({\mathbf{r}})\rangle, \text{ with } \hat{J}({\mathbf{r}}) = \sum_{i < j} J_{ij} u(\mathbf{r}_i, \mathbf{r}_j),$$

where  $J_{ij}$  are optimizable parameters and  $u(\mathbf{r}_i, \mathbf{r}_j)$  polynomials dependent on the electron positions.  $J_{ij}$  optimizable with, e.g. Variational Monte Carlo (VMC)

Hylleras, Z. Phys. 54, 347 (1929); Kutzelnigg, Theoretica chimica acta 68, 445 (1985); Ten-no, J. Chem. Phys. 121, 117 (2004); Jastrow, Phys. Rev. 98, 1479 (1955);

#### Incorporate the Ansatz into the Hamiltonian:

Instead of  $\hat{H} |\Psi\rangle = E |\Psi\rangle$  solve the similarity transformed/transcorrelated (TC) problem

$$\hat{H} |\Psi\rangle = E |\Psi\rangle, \quad \text{with} \quad |\Psi\rangle = e^{\hat{J}} |\Phi\rangle$$

$$e^{-\hat{J}} \rightarrow |\quad \hat{H} e^{\hat{J}} |\Phi\rangle = E e^{\hat{J}} |\Phi\rangle, \quad \left(\hat{J}^{\dagger} = \hat{J}\right)$$

$$\left(e^{-\hat{J}} \hat{H} e^{\hat{J}}\right) |\Phi\rangle = E e^{-\hat{J}} e^{\hat{J}} |\Phi\rangle = E |\Phi\rangle$$

$$|\Phi\rangle$$

$$|\Phi\rangle$$

Baker-Campbell-Hausdorff (BCH) exp. to obtain TC Hamiltonian:

$$\bar{H} = e^{-\hat{J}} \hat{H} e^{\hat{J}} = \hat{H} + [\hat{H}, \hat{J}] + \frac{1}{2!} [[\hat{H}, \hat{J}], \hat{J}] + \cdots^{0}$$

#### For the molecular Hamiltonian the BCH exp. terminates at 2nd order

Hirschfelder, JCP, **39**, 3145 (1963); Boys and Handy, Proc. R. Soc. A (**1969**); **WD**, Luo, Alavi, PRB **99** (7), 075119 (2019); Cohen, Luo, Guther, **WD**, Tew, Alavi, JCP **151** (6), 061101 (2019); **WD**, Cohen, Alavi, Giner, JCP **156** (23), 234108 (2022)

### The Similarity Transformed TC Hamiltonian

#### **Consequences:**

- Transcorrelated  $\bar{H}$  is not Hermitian!  $\rightarrow$  loss of variational principle
- Additional 3-body terms in  $\overline{H}$
- $\rightarrow$  Ansatz-based quantum imaginary time evolution<sup>†</sup> can handle both drawbacks on quantum hardware

#### **Benefits:**

More accurate results with smaller basis sets/less qubits!



<sup>†</sup>McArdle, *et al.*, npj Quantum Information **5**, 75, 2019; McArdle and Tew, arxiv:2006.11181; <sup>\*</sup>Haupt, Hosseini, López Ríos, **WD**, Cohen and Alavi, **arxiv.2302.13683**, 2023

# Applications: Reducing the number of qubits for chemistry

#### Beryllium atom

**Beryllium atom** – exact simulation of a quantum device (no noise) Goal: complete basis set (CBS) limit  $\rightarrow$  full description to compare with experiment



WD, Sokolov, Liao, Lopez Rios, Rahm, Alavi, Tavernelli, arXiv:2303.02007 2023

### ${\bf Lithium\ hydride-LiH}$

Lithium hydride – exact, noiseless simulation of quantum device

"Standard basis sets" not optimized for the TC method  $\rightarrow$  use pre-optimized orbitals, e.g. natural orbitals from perturbation theory calculation (MP2-NOs)



\*CT-F12: Motta et al., Phys. Chem. Chem. Phys. 22, 24270, 2020; <sup>†</sup>Haeffler et al., Phys. Rev. A, 1996, 53, 6, 4127 (1996); WD, Sokolov, Liao, Lopez Rios, Rahm, Alavi, Tavernelli, arXiv:2303.02007 2023

### LiH – Experiment on IBM Quantum devices

Hardware (HW) experiment: lithium hydride dissociation energy on ibm\_lagos



Large effect of noise! Error mitigation techniques<sup>\*</sup> to reduce effect of noise.

WD, Sokolov, Liao, Lopez Rios, Rahm, Alavi, Tavernelli, arXiv:2303.02007 2023; \* Lolur, Skogh, WD, Warren, Biznárová, Osman, Tancredi, Wendin, Bylander, and Rahm, J. Chem. Theory Comput. 2023, 19, 3, 783 Conclusions and outlook

### Conclusion – Transcorrelated Approach on Quantum Hardware

- The **TC method** partially transfers electronic correlations from the wavefunction into the Hamiltonian, **capturing the cusp condition**.
- **Reduce qubit requirements and circuit depth**, due to accurate results with a small basis sets.
- Extends applicability of current and near-term quantum devices to more relevant quantum chemistry problems.



### Workshop – Frontiers of near-term quantum computing



The workshop aims to bring together researchers from the fields of computer science, quantum information and chemistry: https://tinyurl.com/frontiers-of-qc. 29th August – 1st September, 2023, Gothenburg, Sweden

Confirmed speakers:

- Ivano Tavernelli
- Sophia Economou
- Sevag Gharibian
- Richard Kueng

- Xiao Yuan
- Christian Gogolin
- Zoë Holmes
- Stefan Knecht
- Jakob Kottmann

- Panagiotis Barkoutsos
- Ashley Montanaro
- Anand Natarajan
- Pauline Ollitrault
- Benjamin Brown

- Francesco Tacchino
- Juani Bermejo-Vega
- David Muñoz Ramo
- Tony Metger

## Thank you for your attention!

### How to solve non-Hermitian problems on quantum hardware

Since the TC Hamiltonian is **non Hermitian**, VQE not applicable!

 $\rightarrow$  Use Ansatz-based Variational Quantum Imaginary Time Evolution\*



\*McArdle, et al., npj Quantum Information 5, 75, 2019; Sokolov, WD, Luo, Alavi, Tavernelli, arXiv:2201.03049 (2022);

### Hierarchy of methods



Highly accurate methods only applicable to **very small** system sizes. Current quantum computing calculations/experiments use small/**minimal basis sets** far from experimental results, due to **limited number of qubits** 

### Exponential scaling of Full Configuration Interaction

There is a long history and wide variety of computational approaches in chemistry

~				Mol.	#electrons	#states
×K	—	_		$H_2$	2	4
S€S	_	<u> </u>		LiH	4	36
S\$€S	—	~ ┿	$\left( + \sqrt{2} \right)$	$\operatorname{Be}_2$	8	4900
		(		$H_2O$	12	$\sim 8\cdot 10^5$
ЖK	++	<b>`</b> ∔	$\langle +' \rangle + /$	$C_2H_4$	16	$\sim 16\cdot 10^6$
AK	++		$\downarrow \downarrow \downarrow$	$\mathbf{F_2}$	18	$\sim 2\cdot 10^9$
XX	_+	<b>↓</b>				

> 20 electrons  $> \sim 100$  GB information!

 $\rightarrow$  can not even store wavefunction in computer memory

$$\bar{H} = \sum_{pq,\sigma} h_q^p a_{p,\sigma}^{\dagger} a_{q,\sigma} + \frac{1}{2} \sum_{pqrs,\sigma\tau} \bar{V}_{rs}^{pq} a_{p,\sigma}^{\dagger} a_{q,\tau}^{\dagger} a_{s,\tau} a_{r,\sigma} - \frac{1}{6} \sum_{pqrstu,\sigma\tau\lambda} L_{stu}^{pqr} a_{p,\sigma}^{\dagger} a_{q,\tau}^{\dagger} a_{r,\lambda}^{\dagger} a_{u,\lambda} a_{t,\tau} a_{s,\sigma}$$

- Measurement formally scaling as  $N^6$ , with N being the number of orbitals
- Recently shown that N<sup>6</sup>-scaling terms can be neglected to good accuracy\*
- Current work on  $N^4$ -scaling approximation
- Order of magnitude less orbitals: since also no core functions needed in basis set<sup>†</sup>
- Shorter circuit depth, due to more compact ground state!



\* WD et al., Journal of Chemical Physics 156 (23), 234108 (2022); <sup>†</sup>Cohen, Luo, Guther, WD, Tew, Alavi, JCP 151 (6), 061101 (2019);

### Variational Quantum Monte Carlo to optimize Jastrow factors

Minimize variational energy, by optimizing trial wavefunction parameters  $J_{ij}$ :

$$E_{VMC} = \min_{\hat{J}(J_{ij})} \frac{\langle \Phi_0 | e^{\hat{J}} \hat{H} e^{\hat{J}} | \Phi_0 \rangle}{\langle \Phi_0 | e^{2\hat{J}} | \Phi_0 \rangle}, \quad |\Phi_T \rangle = e^{\hat{J}} | \Phi_0 \rangle$$

- The choice of trial wavefunction is critical in VMC calculations  $\rightarrow$  accuracy limited by  $|\Phi_T\rangle = e^{\hat{J}} |\Phi_0\rangle!$
- Hartree-Fock state usually first starting point for  $|\Phi_0\rangle$ , but more elaborate/accurate states possible...
- Polynomial scaling  $\sim N^3$
- Such a VMC calculations to optimize  $J_{ij}$  with a HF state  $|\Phi_0\rangle = |\Phi_{HF}\rangle$  our starting point for the transcorrelated method

#### Explicitly Correlated methods



Linear behavior in electron-electron distance  $r_{ij} = |\mathbf{r}_i - \mathbf{r}_j|$  for small  $r_{ij}$ !

**R12** methods<sup>\*</sup>: 
$$|\Psi\rangle = r_{ij} |\Phi\rangle$$
  
**F12** methods<sup>†</sup>:  $|\Psi\rangle = f(r_{ij}) |\Phi\rangle$ ,  $f(r_{ij}) = \frac{1 - \exp(-\gamma r_{ij})}{\gamma}$   
**Jastrow** Ansatz<sup>‡</sup>:  $|\Psi\rangle = e^{\hat{J}} |\Phi\rangle$ ,  $\hat{J} = \sum_{ij} J_{ij} g(\tilde{r}_{ij})$ 

\* Kutzelnigg, Theoretica chimica acta 68, 445 (1985); <sup>†</sup> Ten-no, J. Chem. Phys. 121, 117 (2004); <sup>‡</sup> Jastrow, Phys. Rev. 98, 1479 (1955);

#### Explicitly Correlated methods



Linear behavior in electron-electron distance  $r_{ij} = |\mathbf{r}_i - \mathbf{r}_j|$  for small  $r_{ij}$ !

 $\begin{aligned} \mathbf{R12} \text{ methods}^* \colon & |\Psi\rangle = r_{ij} |\Phi\rangle \\ \mathbf{F12} \text{ methods}^\dagger \colon & |\Psi\rangle = f(r_{ij}) |\Phi\rangle, \quad f(r_{ij}) = \frac{1 - \exp(-\gamma r_{ij})}{\gamma} \\ \mathbf{Jastrow} \text{ Ansatz}^\dagger \colon & |\Psi\rangle = e^{\hat{J}} |\Phi\rangle, \quad \hat{J} = \sum_{ij} J_{ij} g(\tilde{r}_{ij}) \\ \exp(-x) &\approx 1 - x + \mathcal{O}(x^2), \qquad \tilde{r}_{ij} = \frac{r_{ij}}{1 + r_{ij}}, \quad \lim_{r_{ij} \to 0} \tilde{r}_{ij} \to 0, \lim_{r_{ij} \to \infty} \tilde{r}_{ij} \to 1 \end{aligned}$ 

\* Kutzelnigg, Theoretica chimica acta 68, 445 (1985); † Ten-no, J. Chem. Phys. 121, 117 (2004); ‡ Jastrow, Phys. Rev. 98, 1479 (1955);

Since TC Hamiltonian is **non Hermitian**, variational algorithms like VQE not applicable

$$E_{\text{VQE}} = \min_{\boldsymbol{\theta}} \left\langle \Psi(\boldsymbol{\theta}) \right| \hat{H} \left| \Psi(\boldsymbol{\theta}) \right\rangle$$

Our Approach:

Solve for the **right** eigenvector of non-Hermitian  $\overline{H}$  py **projection** with QITE:

$$|\Phi_0^R\rangle \propto \lim_{t\to\infty} e^{-t\bar{H}} |\phi^R\rangle, \text{ with } \bar{H} |\Phi_0^R\rangle = E |\Phi_0^R\rangle,$$

where  $|\Phi^R\rangle$  is a full expansion in SDs  $|\Phi^R\rangle = \sum_i c_i |D_i\rangle$ 

#### Quantum Imaginary Time Evolution – QITE

 $\rightarrow$  Solve for the **right** eigenvector of non-Hermitian  $\bar{H}$  by (quantum) imaginary-time evolution (QITE)



Motta et al., Nature Physics 16, 205, 2020; \*McArdle, et al., npj Quantum Information 5, 75, 2019;

#### Workflow



\*https://vallico.net/casinoqmc/ <sup>†</sup>https://gitlab.com/kguther/tchint <sup>‡</sup>https://github.com/ghb24/NECI\_STABLE

### (Virtual) orbital optimization



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Unoptimized



- $\rightarrow\,$  include effect of virtuals through orbital optimization / downfolding
- $\rightarrow$  e.g. natural orbitals (NO) from a "cheap" perturbation theory (MP2) calculation



FNO: Sosa et al., Chem. Phys. Lett. 159 (2-3) 148 (1989); Taube and Bartlett Collect. Czech. Chem. Commun. 70, 837 (2005); NISQ: Gonthier et al. arXiv:2012.04001; (2020); Verma et al. JCP 155, 034110 (2021)

### Hydrogen molecule

Favorite quantum chemistry test case: Hydrogen molecule –  $H_2$ CT-F12 approximated explicitly correlated method, by Motta *et al.*\*

Exact statevector simulation – UCCSD Ansatz



\*CT-F12: Motta et al., Phys. Chem. Chem. Phys. 22, 24270, 2020



CT-F12: Motta et al., Phys. Chem. Chem. Phys. 22, 24270, 2020 VQE/MRA+[2]R12: Schleich et al., arXiv:2110.06812, 2021

#### LiH – Dissociation energy

Error statistics and comparison to experimental<sup>\*</sup> dissociation energy



\*Haeffler et al., Phys. Rev. A, 1996, 53, 6, 4127 (1996)

### Spectroscopic Constants



	$H_2$						LiH		
	qubits	$R_e(\text{\AA})$	$D_0(\mathrm{eV})$	$\omega_e(\mathrm{cm}^{-1})$	qubits	$R_e(\text{\AA})$	$D_0(\mathrm{eV})$	$\omega_e(\mathrm{cm}^{-1})$	
no-TC	4	0.73	3.67	4954	12	1.54	2.66	1690	
	8	0.75	3.87	4297	22	1.67	1.80	1283	
	20	0.76	4.19	4353	38	1.62	2.17	1360	
TC	4	0.74	4.69	4435	6	1.60	2.42	1377	
Exp.		0.74	4.52	4401		1.60	2.47	1406	

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### ${\bf LiH-Hardware\text{-}efficient\ Ansatz}$

- LiH at equilibrium bond distance with 3 MP2 NOs.
- Hardware efficient RY Ansatz with linear entangling layer and parity encoding.
- Statevector simulation





#### LiH – Hardware-efficient Ansatz – QASM Simulations

• Reference-state error mitigation  $(REM)^*$  (see Poster session I today - G00/292) or zero-noise extrapolation



 $q_0$ 

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Lolur, Skogh, WD, Warren, Biznárová, Osman, Tancredi, Wendin, Bylander, and Rahm, J. Chem. Theory Comput. 2023, 19, 3, 783